

# Braneworld models with a non-minimally coupled phantom bulk field: a simple way to obtain the $-1$ -crossing at late times

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We investigate general braneworld models, with a non-minimally coupled phantom bulk field and arbitrary brane and bulk matter contents. We show that the effective dark energy of the brane-universe acquires a dynamical nature, as a result of the non-minimal coupling which provides a mechanism for an indirect “bulk-brane interaction” through gravity. For late-time cosmological evolution and without resorting to special ansatzes or to specific areas of the parameter space, we show that the  $-1$ -crossing of its equation-of-state parameter is general and can be easily achieved. As an example we provide a simple, but sufficiently general, approximate analytical solution, that presents the crossing behavior.

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## I. INTRODUCTION

According to cosmological observations our universe is undergoing an accelerating expansion, and the transition to the accelerated phase has been realized in the recent cosmological past [1]. In order to explain this remarkable behavior, and despite the intuition that this can be achieved only through a fundamental theory of nature, we can still propose some paradigms for its description. Thus, we can either consider theories of modified gravity [2], or introduce the concept of dark energy which provides the acceleration mechanism. The dynamical nature of dark energy, at least in an effective level, can originate from various fields, such is a canonical scalar field (quintessence) [3], a phantom field, that is a scalar field with a negative sign of the kinetic term [4], or the combination of quintessence and phantom in a unified model named quintom [5]. The advantage of this combined model is that although in quintessence the dark energy equation-of-state parameter remains always greater than  $-1$  and in phantom cosmology always smaller than  $-1$ , in quintom scenario it can cross  $-1$ .

However, there are strong arguments supporting that the  $-1$ -crossing in a single, minimally-coupled scalar-field model, is unstable under cosmological perturbations realized on trajectories of zero measure [6]. This feature, together with additional theoretical evidences such are quantum-correction incorporation and renormalizability, raised the interest for the investigation of models where the scalar fields are non-minimally coupled to gravity [7].

On the other hand, brane cosmology (motivated by string/M theory), according which our Universe is a brane embedded in a higher dimensional spacetime [8, 9], apart from being closer to a higher-dimensional fundamental theory of nature, it has also great phenomenological successes and a large amount of current research heads towards this direction [10]. The bulk space can contain only a cosmological constant [11], but string theoretical arguments led to the insertion of bulk scalar fields, since at the level of the low-energy  $5D$  theory it is natural to expect the appearance of a dilaton-like scalar field in addition to the Einstein-Hilbert action [12]. Therefore, the cosmological evolution on the brane-universe is a combined effect of both the brane matter content and the bulk scalar field [13, 14, 15, 16, 17, 18]. In addition, being closer to scalar-tensor gravity theories and to the theoretical evidences described above, one can consider the bulk scalar field to be non-minimally coupled to the  $5D$  curvature scalar [19].

In the present work we are interested in investigating  $5D$  braneworld models with a non-minimally coupled phantom bulk field. This scenario combines the advantages of  $4D$  phantom cosmology, together with the advantages of non-minimally coupled  $4D$  or brane cosmology. In particular, the basic question is whether we can acquire an effective cosmological evolution on the brane-universe, where the dark energy equation-of-state parameter can cross the phantom divide in the near cosmological past, without choosing special ansatzes for the various quantities, or moving

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in specific, small areas of the parameter space. Since it is already known that in some special cases, non-minimally coupled in conventional 4D cosmology can lead even single phantom-field models to experience the  $-1$ -crossing [20], it is interesting to see if this behavior can be preserved in (closer to a multi-dimensional theory of nature) brane cosmology. Indeed, it proves that such models not only do present the aforementioned behavior, in agreement with observations, but they do so for a large solution sub-class and without the need of restricting to a specific area of the parameter space. In particular, under the low-energy (late-time) assumptions and up to first order in terms of the non-minimal coupling parameter, the  $-1$ -crossing can be simply acquired if the term  $\phi\dot{\phi}$  on the brane-universe (where  $\phi$  is the phantom bulk field) changes sign.

However, we have to make a comment about the quantum behavior of the examined model. As it is known, the discussion about the construction of quantum field theory of phantoms is still open in the literature. For instance in [21] the authors reveal the causality and stability problems and the possible spontaneous breakdown of the vacuum into phantoms and conventional particles in four dimensions, arising from the energy negativity (if one desires to maintain the unitarity of the theory). On the other hand, there have also been serious attempts in overcoming these difficulties and construct a phantom theory consistent with the basic requirements of quantum field theory [22], with the phantom fields arising as an effective description. Since every warped higher dimensional model is reduced to an effective 4D one in low energies, the aforementioned discussion concerns the 5D phantom scenario, too. The present analysis is just a first approach on the  $-1$ -crossing in phantom braneworlds. Definitely, the subject of quantization of such models is open and needs further investigation.

The plan of the work is as follows: In section II we formulate general braneworld models with a non-minimally coupled phantom bulk field and arbitrary brane and bulk matter contents. In section III we provide the general late-time evolution on the brane-universe and we present the main result of our investigation, namely the relation between the brane-universe's effective equation-of-state parameter with the values of the bulk field and its derivative on the brane. In section IV we construct a simple but quite general approximate analytical solution, which experiences the  $-1$ -crossing. Finally, section V is devoted to summarize our results.

## II. THE MODEL

We consider a general class of single-brane models, in the presence of a non-minimally coupled phantom bulk field, characterized by the action:

$$\mathcal{S} = \int d^4x dy \sqrt{-G} \left\{ f(\phi) \mathcal{R} - \Lambda + \frac{1}{2} (\nabla \phi)^2 - V(\phi) - \mathcal{L}_B^{(m)} \right\} + \int d^4x \sqrt{-g} \left\{ -\sigma + \mathcal{L}_b^{(m)} \right\}. \quad (1)$$

$G_{MN}$  describes the 5D metric while  $g$  denotes the associated induced metric on the brane, located at  $y = 0$  without loss of generality, and as usual  $\Lambda$  is the 5D cosmological constant and  $\sigma$  is the brane-tension. The term  $\mathcal{L}_B^{(m)}$  stands for possible forms of bulk matter apart from the phantom field, while  $\mathcal{L}_b^{(m)}$  accounts for the matter content of the brane-universe. For the moment we consider the coupling  $f(\phi)$ , to the 5D Ricci scalar  $\mathcal{R}$ , to be an arbitrary, general function.

The Einstein's equations arise by variation of the action:

$$f(\phi) \left( \mathcal{R}_{MN} - \frac{1}{2} G_{MN} \mathcal{R} \right) - \nabla_M \nabla_N f(\phi) + G_{MN} \nabla^2 f(\phi) = \frac{1}{2} T_{MN}, \quad (2)$$

where  $T_{MN}$  is the total energy-momentum tensor:

$$T_{MN} = T_{MN}^{(\phi)} + T_{MN}^{(B)} + T_{MN}^{(b)} - G_{MN} \Lambda - G_{\mu\nu} \delta_M^\mu \delta_N^\nu \sigma \delta(y). \quad (3)$$

In the expression above,  $T_{MN}^{(\phi)}$  accounts for the phantom bulk field

$$T_{MN}^{(\phi)} = -\nabla_M \phi \nabla_N \phi + G_{MN} \left( \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right). \quad (4)$$

Similarly,  $T_{MN}^{(B)}$  stands for the bulk part of the energy-momentum tensor, while  $T_{MN}^{(b)}$  accounts for the brane-matter content. Finally, varying the action in terms of the phantom field, we obtain its evolution equation:

$$-\nabla^2 \phi - \frac{dV}{d\phi} + \mathcal{R} \frac{df}{d\phi} - \frac{d\sigma}{d\phi} \delta(y) = 0. \quad (5)$$

For the metric we will use the following form [23]:

$$ds^2 = -n^2(y, t)dt^2 + a^2(y, t)\gamma_{ij}dx^i dx^j + b^2(y, t)dy^2, \quad (6)$$

which corresponds to a maximally symmetric, induced Friedmann-Robertson-Walker geometry on the brane. Thus, the isometry assumption along three dimensional  $\mathbf{x}$ -slices, including the brane, allows us to consider the phantom bulk field depending only on the fifth coordinate. Furthermore, using square brackets to denote the jump of any quantity across the brane ( $[Q] \equiv Q(y_+) - Q(y_-)$ ), and assuming  $S^1/\mathbb{Z}_2$  symmetry across it, we restrict our interest only in the  $[0, +\infty)$  interval, obtaining

$$[Q'] = 2Q'(0^+). \quad (7)$$

This relation is going to be used in the elaboration of the discontinuities of the derivatives of various quantities at the location of the brane [24].

In these coordinates, the phantom energy-momentum tensor writes:

$$T_{MN}^{(\phi)} = \begin{pmatrix} -\frac{1}{2}\dot{\phi}^2 - \frac{n^2}{2b^2}\phi'^2 + n^2V & 0 & -\dot{\phi}\phi' \\ 0 & a^2\gamma_{ij}\left[-\frac{1}{2n^2}\dot{\phi}^2 + \frac{1}{2b^2}\phi'^2 - V\right] & 0 \\ -\dot{\phi}\phi' & 0 & -\frac{1}{2}\phi'^2 - \frac{b^2}{2n^2}\dot{\phi}^2 - b^2V \end{pmatrix}, \quad (8)$$

where primes and dots denote derivatives with respect to  $y$  and  $t$  respectively. For the brane-universe we assume that it contains a perfect fluid with equation of state  $p = w_m\rho$ , where  $\rho$  and  $p$  are its energy density and pressure respectively. Thus, the brane energy-momentum tensor reads:

$$T_{MN}^{(b)} = \frac{\delta(y)}{b} \begin{pmatrix} \rho n^2 & 0 & 0 \\ 0 & pa^2\gamma_{ij} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (9)$$

Similarly, assuming an ideal fluid for the arbitrary forms of bulk matter, with energy density  $\rho_B$  and pressures  $P_B$  and  $\overline{P}_B$  (since the pressure on the fifth dimension can be different), we acquire:

$$T_{MN}^{(B)} = \begin{pmatrix} \rho_B n^2 & 0 & -n^2 P_5 \\ 0 & P_B a^2 \gamma_{ij} & 0 \\ -n^2 P_5 & 0 & \overline{P}_B b^2 \end{pmatrix}. \quad (10)$$

Note that we have allowed for an energy-exchange function  $P_5$  between the bulk and the brane-universe [16]. Finally, all quantities are considered as functions of  $y$  and  $t$ , apart from the brane-fluid's  $\rho$  and  $p$  which depend only on time.

In order to focus on the cosmological evolution on the brane we use the Gaussian normal coordinates ( $b(y, t) = 1$ ) [14]. Thus, the non-trivial five-dimensional Einstein equations consist of three dynamical:

$$3 \left\{ \frac{a'}{a} \left( \frac{a'}{a} + \frac{n'}{n} \right) - \frac{1}{n^2} \left( \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) + \frac{\ddot{a}}{a} \right) - \frac{k}{a^2} \right\} f - \frac{1}{n^2} \left\{ \ddot{f} + \left( 3\frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) \dot{f} \right\} + \left( 3\frac{a'}{a} + \frac{n'}{n} \right) f' = -\frac{1}{4}\phi'^2 - \frac{1}{4n^2}\dot{\phi}^2 - \frac{1}{2}V(\phi) - \frac{1}{2}\Lambda + \frac{1}{2}\overline{P}_B, \quad (11)$$

$$a^2\gamma_{ij} \left\{ \frac{a'}{a} \left( \frac{a'}{a} + 2\frac{n'}{n} \right) + 2\frac{a''}{a} + \frac{n''}{n} \right\} f + \frac{a^2}{n^2}\gamma_{ij} \left\{ \frac{\dot{a}}{a} \left( -\frac{\dot{a}}{a} + 2\frac{\dot{n}}{n} \right) - 2\frac{\ddot{a}}{a} \right\} f - kf\gamma_{ij} + \gamma_{ij} \left\{ -\frac{a^2}{n^2} \left[ \ddot{f} + \left( 2\frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) \dot{f} \right] + a^2 \left[ f'' + \left( 2\frac{a'}{a} + \frac{n'}{n} \right) f' \right] \right\} = \frac{a^2}{2}\gamma_{ij} \left[ -\frac{1}{2n^2}\dot{\phi}^2 + \phi'^2 - V(\phi) \right] + \frac{a^2}{2}\gamma_{ij}(P_B - \Lambda) + \frac{a^2}{2}\gamma_{ij}\delta(y)(p - \sigma(\phi)), \quad (12)$$

$$\ddot{\phi} + \left( 3\frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) \dot{\phi} - n^2 \left\{ \phi'' + \left( \frac{n'}{n} + 3\frac{a'}{a} \right) \phi' \right\} - n^2 \frac{dV(\phi)}{d\phi} + n^2 \mathcal{R} f' - n^2 \frac{d\sigma(\phi)}{d\phi} \delta(y) = 0, \quad (13)$$

and two constraint equations:

$$3 \left\{ \left( \frac{\dot{a}}{a} \right)^2 - n^2 \left( \frac{a''}{a} + \left( \frac{a'}{a} \right)^2 \right) + k\frac{n^2}{a^2} \right\} f - n^2 \left\{ f'' + 3\frac{a'}{a} f' \right\} + 3\frac{\dot{a}}{a} \dot{f} = -\frac{1}{4}\dot{\phi}^2 - \frac{n^2}{4}\phi'^2 + \frac{n^2}{2}V(\phi) + \frac{n^2}{2}\rho_B + \frac{n^2}{2}\delta(y)\rho + \frac{n^2}{2}\delta(y)\sigma(\phi) + \frac{n^2}{2}\Lambda, \quad (14)$$

$$3 \left( \frac{n'}{n} \frac{\dot{a}}{a} - \frac{\dot{a}'}{a} \right) f - \dot{f}' + \frac{n'}{n} \dot{f} = -\frac{1}{2} \dot{\phi} \phi' - \frac{n^2}{2} P_5. \quad (15)$$

Note that we have allowed for a  $\phi$ -dependence of the brane tension  $\sigma(\phi)$ . Finally, as usual the 5D Ricci scalar  $\mathcal{R}$  is given by

$$\mathcal{R} = 3 \frac{k}{a^2} + \frac{1}{n^2} \left\{ 6 \frac{\ddot{a}}{a} + 6 \left( \frac{\dot{a}}{a} \right)^2 - 6 \frac{\dot{a} \dot{n}}{a n} \right\} - 6 \frac{a''}{a} - 2 \frac{n''}{n} - 6 \left( \frac{a'}{a} \right)^2 - 6 \frac{a' n'}{a n}. \quad (16)$$

In order to obtain the boundary conditions for the aforementioned cosmological system, we integrate the 00 and  $ii$  components of the 5D Einstein equations around the brane, making use of (7). Thus, we result to the following junction (Israel) conditions (setting also  $n(0, t) = 1$  without loss of generality):

$$-6f \frac{a'_0}{a_0} + 2 \left. \frac{df}{d\phi} \right|_0 \phi'_0 = \frac{1}{2}(\rho + \sigma) \quad (17)$$

$$4f \frac{a'_0}{a_0} + 2n'_0 f - 2 \left. \frac{df}{d\phi} \right|_0 \phi'_0 = \frac{1}{2}(p - \sigma) \quad (18)$$

$$2\phi'_0 + 4 \left. \frac{df}{d\phi} \right|_0 \left( n'_0 + 3 \frac{a'_0}{a_0} \right) = - \left. \frac{d\sigma}{d\phi} \right|_0, \quad (19)$$

where the index 0 denotes the values of the corresponding quantities at the location of the brane. For notation simplification in the following we omit it, i.e  $a$ ,  $n$  and  $\phi$  and their derivatives, stand for the corresponding quantities on the brane. Furthermore, we call  $f_\phi$  and  $f_\sigma$  the terms  $\left. \frac{df}{d\phi} \right|_0$  and  $\left. \frac{d\sigma}{d\phi} \right|_0$  respectively.

Finally, equations (17) can be re-written as:

$$\frac{a'}{a} = -\frac{1}{2} \frac{(\sigma + 2f_\phi f_\sigma)}{u} + \frac{1}{8} \frac{(3p - \rho)}{u} - \frac{1}{16} \frac{(\rho + p)}{f}, \quad (20)$$

$$n' = -\frac{1}{2} \frac{(\sigma + 2f_\phi f_\sigma)}{u} + \frac{1}{8} \frac{(3p - \rho)}{u} + \frac{3}{16} \frac{(\rho + p)}{f}, \quad (21)$$

$$\phi' = -f_\phi \frac{(3p - \rho)}{u} - \frac{(3f f_\sigma - 4\sigma f_\phi)}{u}, \quad (22)$$

where we have defined

$$u \equiv 6f + 16f_\phi^2. \quad (23)$$

### III. GENERAL LATE-TIME COSMOLOGICAL EVOLUTION ON THE BRANE-UNIVERSE

In this section we develop the formalism for the late-time brane-evolution investigation, following [15, 18]. First of all, using the boundary conditions (17)-(19), together with (15) at  $y = 0$ , we acquire the energy conservation equation on the brane:

$$\dot{\rho} + 3 \frac{\dot{a}}{a} (\rho + p) + 2P_5 = 0, \quad (24)$$

which in the case  $P_5 \neq 0$  describes the direct bulk-brane energy flow, which can have both signs. Similarly, using equation (11) at  $y = 0$ , we obtain the cosmological evolution on the brane:

$$3f \left[ \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} \right] + \ddot{f} + 3 \frac{\dot{a}}{a} \dot{f} - \frac{1}{4} (\dot{\phi})^2 - \frac{1}{2} (V + \Lambda) = \frac{1}{64u} (3p - \rho - 4\sigma)^2 - \frac{6}{(16)^2} \frac{(\rho + p)^2}{f} - \frac{1}{2} \overline{P}_B + \frac{f_\phi f_\sigma}{4u} (3p - \rho - 4\sigma) - \frac{3f}{8u} (f_\sigma)^2. \quad (25)$$

Finally, the use of (13) at  $y = 0$  provides the phantom field evolution on the brane:

$$-\ddot{\phi} - 3 \left( \frac{\dot{a}}{a} \right) \dot{\phi} + \frac{dV}{d\phi} - 6f_\phi \left[ \frac{k}{2a^2} + \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 \right] + \phi' \left( n' + 3 \frac{a'}{a} \right) + 6f_\phi \left( \frac{a'}{a} \right) \left( \frac{a'}{a} + n' \right) = -\hat{\phi}'' - 2f_\phi \left( 3 \frac{\hat{a}''}{a} + \hat{n}'' \right). \quad (26)$$

The terms  $\hat{\phi}''(t), \hat{a}''(t), \hat{n}''(t)$  denote the unknown, “non-distributional” parts of the corresponding derivatives [18]. Although they could be set to zero, we consider them to depend on the structure of the bulk.  $\hat{a}''$  and  $\hat{n}''$  can be expressed in terms of  $\hat{\phi}''$  and the standard quantities, i.e.  $a(t), \phi(t)$ , their time-derivatives and the various matter densities. Assuming  $f_\sigma = 0$  and following [18], the corresponding elimination leads to the evolution equations on the brane:

$$\frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} \approx \frac{\sigma^2}{12fu} + \frac{\sigma}{24fu} (\rho - 3p) - \frac{1}{6f} \overline{P}_B + \frac{1}{6f} (V + \Lambda) + \frac{\dot{\phi}^2}{12f} (1 + 4f_{\phi\phi}) + \frac{f_\phi}{3f} \left( \ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} \right), \quad (27)$$

$$\begin{aligned} & -\ddot{\phi} - 3 \frac{\dot{a}}{a} \dot{\phi} + \hat{\phi}'' - \frac{u_\phi}{2u} \dot{\phi}^2 \approx \\ & \approx \frac{3k}{a^2} f_\phi - \frac{6f}{u} \frac{dV}{d\phi} - \frac{2f_\phi}{u} (3P_B + \overline{P}_B - \rho_B) + 10 \frac{f_\phi}{u} (V + \Lambda) + 4 \frac{f_\phi (u + f_\phi u_\phi) \sigma}{u^3} (\rho - 3p) + \frac{8f_\phi (u + f_\phi u_\phi) \sigma^2}{u^3}, \end{aligned} \quad (28)$$

where  $u_\phi = \left. \frac{du}{d\phi} \right|_0$  and  $f_{\phi\phi} = \left. \frac{d^2 f}{d\phi^2} \right|_0$ . We mention that in order to simplify the above two equations we have focused on the late-time approximation, namely neglecting  $\rho^2$ -terms [15, 25] since late time is equivalent to low-energy. Finally, note that the phantom evolution equation still contains the unknown function  $\hat{\phi}''$ . In order to provide a specific analytical solution sub-class we have to use an ansatz for it, and this will be done in the simple example of the next section.

Equations (24), (27) and (28) describe the cosmological evolution of a brane-universe with arbitrary curvature, in the case of arbitrary bulk and brane contents, for a general coupling  $f(\phi)$ , in the low-energy approximation. In order to proceed we have to make a choice for the form of  $f(\phi)$ . We choose the well-studied quadratic form:

$$f(\phi) = M_5^3 \left( 1 - \frac{\xi}{2} \phi^2 \right), \quad (29)$$

where  $\xi$  is the coupling parameter [19], and  $M_5$  the 5D Planck mass. This ansatz is a good approximation to a general coupling function for small phantom field values. Using (29) we obtain:  $f_\phi = -M_5^3 \xi \phi$  and  $u = 6M_5^3 \left( 1 - \frac{\xi}{2} \phi^2 \right) + 16M_5^6 \xi^2 \phi^2$ .

As a next step we proceed to the dark energy field formulation [15], introducing the dark energy field  $\chi(t)$  in a way that the second-order equation (27) is replaced by two first-order ones:

$$\left( \frac{\dot{a}}{a} \right)^2 = -\frac{k}{a^2} + 2\gamma\rho + \chi + \lambda, \quad (30)$$

$$\dot{\chi} + 4 \frac{\dot{a}}{a} \left\{ \chi + \frac{1}{12f} \left[ \overline{P}_B - \frac{1}{2} (1 - 4\xi) \dot{\phi}^2 - V + 2\xi\phi \left( \ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} \right) \right] \right\} = 4\gamma P_5 - 2\dot{\gamma}\rho - \dot{\lambda}, \quad (31)$$

where  $\beta, \gamma$  and  $\lambda$  are given from

$$\beta \equiv \frac{1}{24fu} \quad (32)$$

$$\gamma \equiv \sigma\beta \quad (33)$$

$$\lambda \equiv \frac{1}{12f} \left( \Lambda + \frac{\sigma^2}{2u} \right). \quad (34)$$

A simple verification of the above formulation can be obtained by differentiating (30), substituting in (31) and using (24). In this way the initial equation (27) can be recovered.

Under the constructed formulation, equation (30) corresponds to the conventional  $4D$  Friedmann equation of the (brane)-universe, and the function  $\lambda$  is just the effective  $4D$  cosmological constant of the brane. In the minimal coupling case, i.e. when  $\xi = 0$ , we obtain  $\lambda = \frac{1}{12M_5^3} \left( \Lambda + \frac{\sigma^2}{12M_5^3} \right) = \text{const}$  (i.e.  $\dot{\lambda} = 0$ ) and thus the universe's dark energy behaves like a cosmological constant. In addition, one can fine-tune  $\lambda$  to zero [9] as:

$$\lambda = \frac{1}{12M_5^3} \left( \Lambda + \frac{\sigma^2}{12M_5^3} \right) = 0, \quad (35)$$

which corresponds to the familiar Randall-Sundrum fine-tuning and to the known brane solution with no dark energy.

In the non-minimally coupling case we observe that  $\lambda$  is not a constant anymore but it has acquired a dynamical nature ( $\dot{\lambda} \neq 0$ ) due to the dynamical nature of the coupling  $f(\phi(t))$ . Therefore,  $\lambda$  corresponds to the effective  $4D$  dark energy of the brane-universe. Surprisingly enough one can acquire an analytical expression for its equation-of-state parameter  $w_{eff}$ , up to second order corrections in terms of the coupling  $\xi$ . In particular, differentiating (34), using the ansatz (29) and keeping only first order  $\xi$ -terms we obtain:

$$\dot{\lambda} - \xi \phi \dot{\phi} \lambda = 0 \quad \Rightarrow \quad \dot{\lambda} + 3 \frac{\dot{a}}{a} \left( -\frac{1}{3} \xi \phi \dot{\phi} \frac{a}{\dot{a}} \right) \lambda = 0. \quad (36)$$

This relation is independent of  $\lambda$  and  $\xi$  normalizations (since some authors consider  $2M_5^3$  instead of  $M_5^3$ ) and thus general. Therefore, we can straightforwardly make the identification:

$$w_{eff} = -1 - \frac{1}{3} \xi \phi \dot{\phi} \frac{a}{\dot{a}}. \quad (37)$$

Relation (37) is the main result of the present work, and provides the equation-of-state parameter for the dark energy of a brane-universe, embedded into a bulk with a non-minimally coupled phantom field and arbitrary bulk and brane matter contents, up to first order in terms of the coupling  $\xi$ . In the minimally-coupled case ( $\xi = 0$ ) we acquire  $w_{eff} = -1$ , that is we verify that the dark energy is simply just a cosmological constant. However, in the non-minimally coupled model, which is the case of interest of the present work, we observe that the obtained cosmological behavior is very interesting. In particular, if  $\phi \dot{\phi}$  is negative, then  $-1 < w_{eff}$ , that is the universe's dark energy behaves like quintessence. It is interesting to notice that we obtain an effective  $4D$  quintessence behavior, although we have the presence of a phantom bulk field. On the other hand, if  $\phi \dot{\phi}$  is positive, then  $w_{eff} < -1$  and dark energy behaves like a  $4D$  phantom.

The most interesting case is when  $\phi \dot{\phi}$  changes sign during the cosmological evolution. In this scenario  $w_{eff}$  crosses the phantom divide. In addition, if initially we have  $\phi \dot{\phi} < 0$  and after some particular moment we have  $\phi \dot{\phi} > 0$ , then the  $-1$ -crossing can take place from above to below, that is consistently with observations. We mention that the crossing can be achieved with the use of only one bulk field, thus the constructed model is quite economical. This feature was already known to hold in  $4D$  single-field, non-minimally coupled cosmology, but only in specific cases [20]. However, the non-trivial fact that it is maintained in the higher-dimensional, and thus closer to a fundamental theory of nature, brane cosmology, and without the need of restricting to specific, small areas of the parameter space, provides an additional argument for the significance of the non-minimal coupling for the description of nature. Finally, note that the dynamical nature of the  $4D$  effective dark energy and the possible brane-universe acceleration, is obtained independently from the possible direct energy flow between the bulk and the brane. In other words, the non-minimal coupling provides a mechanism for an indirect “bulk-brane interaction”, through gravity.

In conclusion, in this section we have formulated the general late-time (low-energy) cosmological evolution on the brane-universe. In order to acquire a specific example one can either solve the aforementioned equations numerically or choose some simple ansatzes for the scale factor, the bulk field and the matter terms, and solve the equations analytically. In the next section, we use some general but simple ansatzes, in order to present more transparently the cosmological implications of the constructed model.

#### IV. CROSSING $-1$ : A SIMPLE APPROXIMATE SOLUTION

Let us now present a simple, but sufficiently general, approximate, analytical cosmological solution. Since we focus our analysis at late times, it is reasonable to consider the widely-used power-law ansatzes. Thus, for the bulk field we assume:

$$\phi(t) = \frac{C_\alpha}{t^\alpha} + \frac{C_\beta}{t^\beta}, \quad (38)$$

with  $C_\alpha, C_\beta$  being constants. Similarly, for the scale factor we can safely consider the solution:

$$a(t) = C_3 t^{\nu_1}. \quad (39)$$

Now we have to choose the exchange term  $P_5$  that is present in (24), as well as the bulk pressure  $\bar{P}_B$  that appears in (25), which are both functions of time, corresponding to the values of  $\bar{P}_B(y, t)$  and  $P_5(y, t)$  on the brane. The energy-momentum conservation  $\nabla_M T_N^M = 0$  cannot fully determine  $\bar{P}_B$  and  $P_5$  and a particular model of the bulk matter is required. Although we could consider the ansatz [16] (see also [26]):

$$P_5(t) = C_5 \left[ \frac{\dot{a}(t)}{a(t)} \right] a(t)^{\nu_2}, \quad (40)$$

for the investigation of this section and for simplification of the results it is adequate to set  $P_5 = 0$ . For the bulk pressure we assume  $\bar{P}_B = \text{const}$ , and the constant can be absorbed in a re-definition of the bulk potential. This can be chosen as [17]:

$$V(\phi) = \frac{\mu^2}{2} \phi^2, \quad (41)$$

which is a general form, consistent with the brane stabilization mechanism [27]. Finally, for the matter content of the universe we have already assumed an ideal fluid, with energy density and pressure connected by:

$$p = w_m \rho. \quad (42)$$

Thus, using (42) we can easily solve (24) obtaining:

$$\rho = \frac{C_\rho}{a^{3(1+w_m)}}, \quad (43)$$

with  $a(t)$  given by (39).

We mention that the considered ansatzes for  $a(t)$ ,  $P_5$ ,  $\bar{P}_B$  and  $V(\phi)$  are not crucial for the obtained results, and they are chosen just for presentation reasons. The only necessary behavior is a  $\phi(t)$  with a derivative that changes sign during the evolution, as we have discussed in the previous section. The ansatz (38) can fulfill this conditions in a simple way, but any other ansatz with a derivative sign-flip could be equivalently used. Finally, if one use an arbitrary ansatz for  $\phi$  without a derivative sign-flip, then according to (37) he will acquire a  $w_{eff}$  lying always on the same side of the phantom divide during cosmological evolution.

The final step is the substitution of the aforementioned ansatzes, together with (43), into the cosmological equations (30) and (31). In order to simplify the calculations we consider only the flat brane-universe case ( $k = 0$ ), and since we are investigating the late-time behavior we only keep terms up to  $\mathcal{O}(t^{-2\alpha})$  and  $\mathcal{O}(t^{-2\beta})$  [18]. Finally, as stated in section III, we keep terms up to  $\mathcal{O}(\xi^2)$ . Therefore, within these approximations, the two cosmological equations become:

$$\frac{\Lambda}{6M_5^3} + \frac{\sigma^2}{72M_5^6} - \frac{\nu_1(2\nu_1 - 1)}{t^2} + \frac{(1 - 3w_m)\sigma C_\rho t^{-3(1+w_m)\nu_1}}{144M_5^6} + \mathcal{O}(t^{-2\alpha}) + \mathcal{O}(t^{-2\beta}) + \mathcal{O}(\xi^2) = 0, \quad (44)$$

$$\begin{aligned} & -\hat{\phi}'' + t^{-2-\alpha} \left[ C_\alpha \alpha(\alpha + 1 - 3\nu_1) - \frac{\sigma(1 - 3w_m)}{9M_5^3} C_\rho C_\alpha \xi \right] + t^{-2-\beta} \left[ C_\beta \beta(\beta + 1 - 3\nu_1) - \frac{\sigma(1 - 3w_m)}{9M_5^3} C_\rho C_\beta \xi \right] - \\ & -t^{-\alpha} \left[ C_\alpha \mu^2 + \frac{5}{3} C_\alpha \Lambda \xi + \frac{2C_\alpha \sigma^2}{9M_5^3} \xi \right] - t^{-\beta} \left[ C_\beta \mu^2 + \frac{5}{3} C_\beta \Lambda \xi + \frac{2C_\beta \sigma^2}{9M_5^3} \xi \right] + \mathcal{O}(t^{-2\alpha}) + \mathcal{O}(t^{-2\beta}) + \mathcal{O}(\xi^2) = 0. \end{aligned} \quad (45)$$

A last assumption concerns the unknown function  $\hat{\phi}''$ . Although its specific form is not important, in order to simplify the calculations and following [18] we consider:

$$\hat{\phi}'' = \frac{C_{\phi\alpha}}{t^{\alpha+2}} + \frac{C_{\phi\beta}}{t^{\beta+2}}. \quad (46)$$

In order for equation (44) to be satisfied for all  $t$  we require:

$$\nu_1 = \frac{2}{3(1 + w_m)} \quad (47)$$

$$\Lambda + \frac{\sigma^2}{12M_5^3} = 0 \quad (48)$$

$$\nu_1(2\nu_1 - 1) = \frac{\sigma(1 - 3w_m)}{144M_5^6} C_\rho. \quad (49)$$

Note that (48) is the standard Randall-Sundrum fine-tuning for a vanishing cosmological constant on the brane, in the minimally-coupling case in the absence of matter. Under these conditions, the requirement of satisfaction of (45) at all times leads to:

$$\mu^2 = -\frac{\xi\sigma^2}{12M_5^3} \quad (50)$$

$$C_\alpha \alpha(\alpha + 1 - 3\nu_1) - 16M_5^3 C_\alpha \nu_1(2\nu_1 - 1)\xi - C_{\phi\alpha} = 0 \quad (51)$$

$$C_\beta \beta(\beta + 1 - 3\nu_1) - 16M_5^3 C_\beta \nu_1(2\nu_1 - 1)\xi - C_{\phi\beta} = 0. \quad (52)$$

Conditions (47)-(52) are necessary in order for the selected ansatzes to form a self-consistent cosmological solution. The quadratic equations (51) and (52) lead to:

$$\alpha = \frac{3\nu_1 - 1}{2} \pm \sqrt{\frac{(3\nu_1 - 1)^2}{4} + \frac{C_{\phi\alpha}}{C_\alpha} + 16M_5^3 [\nu_1(2\nu_1 - 1)] \xi} \quad (53)$$

$$\beta = \frac{3\nu_1 - 1}{2} \pm \sqrt{\frac{(3\nu_1 - 1)^2}{4} + \frac{C_{\phi\beta}}{C_\beta} + 16M_5^3 [\nu_1(2\nu_1 - 1)] \xi}. \quad (54)$$

As we can see, if  $\nu_1 \geq 1/2$ , that is according to (47) if  $w_m \leq 1/3$ , the exponents  $\alpha$  and  $\beta$  are real for any value of  $\xi$ . On the other hand for  $\nu_1 < 1/2$  ( i.e  $w_m > 1/3$ ) the condition  $\alpha, \beta \in \mathbb{R}$  leads to a restriction in  $\xi$ -values, namely:

$$\xi \leq \min(\xi_1, \xi_2), \quad (55)$$

where

$$\xi_1 = \xi_c \frac{\left[ (3\nu_1 - 1)^2 + 4 \frac{C_{\phi\alpha}}{C_\alpha} \right]}{12\nu_1(1 - 2\nu_1)} \quad (56)$$

$$\xi_2 = \xi_c \frac{\left[ (3\nu_1 - 1)^2 + 4 \frac{C_{\phi\beta}}{C_\beta} \right]}{12\nu_1(1 - 2\nu_1)}. \quad (57)$$

In these expressions we have used the definition  $\xi_c \equiv \frac{3}{16M_5^3}$ , namely the conformal value of the non-minimal coupling parameter  $\xi$ . Therefore, in the (non-conventional) case  $w_m > 1/3$ , the value of  $\xi$  must be sufficiently small.

Let us now examine the cosmological behavior of the simple, approximate, but quite general, model of this section. According to (37), and using the ansatzes (38), (39) we obtain:

$$w_{eff} = -1 + \frac{1}{3\nu_1} \left( \frac{C_\alpha}{t^\alpha} + \frac{C_\beta}{t^\beta} \right) \left( \frac{\alpha C_\alpha}{t^\alpha} + \frac{\beta C_\beta}{t^\beta} \right) \xi, \quad (58)$$

with  $\alpha$  and  $\beta$  given by (53) and (54). Note that due to the parameter eliminations through conditions (47)-(52), one can either impose a value for  $w_m$  and acquire the value of  $C_\rho$ , or the opposite. Since the direct observable is  $w_m$  we prefer to use its value as an input, and thus eliminate  $C_\rho$  from  $w_{eff}$ 's calculation.

As we have already mentioned, for  $\xi = 0$  dark energy behaves like a cosmological constant ( $w_{eff} = -1 = const$ ), but the non-minimal coupling gives it a dynamical nature. Assuming  $\phi(t) \geq 0$  (i.e. choosing  $C_\alpha, C_\beta > 0$ ), then if  $\dot{\phi}(t)$  changes sign during the cosmological evolution then  $w_{eff}$  crosses the phantom divide  $-1$ , while if  $\dot{\phi}(t)$  preserves the same sign,  $w_{eff}$  lies always on one side of the phantom divide. However, one could consider cases where  $\phi(t)$  changes sign or where both  $\phi(t)$  and  $\dot{\phi}(t)$  do. Finally, as can be seen from (58) and (53),(54) the decisive role for the determination of  $w_{eff}$  is played by the  $\phi$ -parameters ( $C_\alpha, C_\beta$ ),  $w_m$  and of course  $\xi$ , with the rest parameters being non-relevant (for realistic choices).

In order to acquire a better comparison with the observational data, in the following we provide the evolution of  $w_{eff}$  versus the redshift  $z$ , given by  $(1+z) = a_0/a(t)$  with  $a_0$  the present value, where we fix  $C_3$  in order to acquire  $a_0 = 1$ . Furthermore, for the equation-of-state parameter of the matter content of the universe we use the value  $w_m = 1/3$ , since a relativistic matter is the most natural choice. This value for  $w_m$  has an additional advantage, namely it leads to  $\nu_1 = 1/2$  (see (47)) and thus  $M_5$  is eliminated from the final results (see (53) and (54)). Therefore, since  $M_5$  defines the units of the present model, its disappearance from the final relations ((47), (53), (54)) and (58)) allows us to choose the units at will. Thus, we choose the units in order for the parameters  $C_\alpha, C_{\phi\alpha}, C_\beta, C_{\phi\beta}$  to be of order 1.



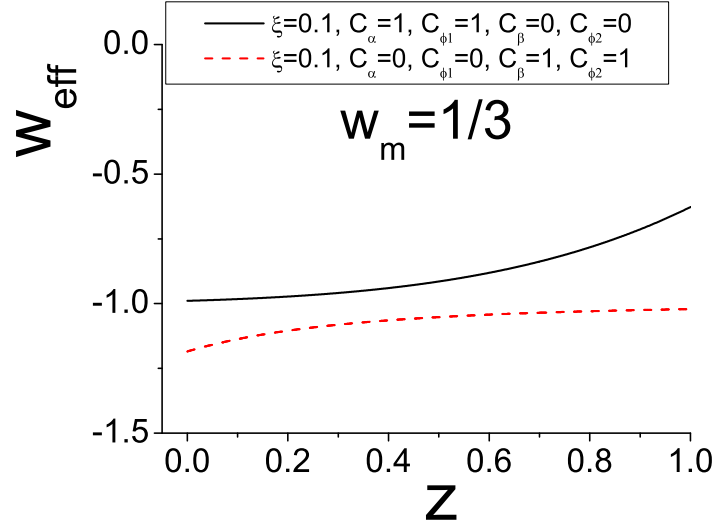


FIG. 1: (Color online)  $w_{eff}$  versus  $z$ , for  $w_m = 1/3$  and  $\xi = 0.1$ . The upper curve corresponds to  $C_\alpha = 1, C_{\phi\alpha} = 1, C_\beta = 0, C_{\phi\beta} = 0$ , while the lower one to  $C_\alpha = 0, C_{\phi\alpha} = 0, C_\beta = 1, C_{\phi\beta} = 1$ .

In fig. 1 we depict  $w_{eff}$  versus  $z$  for  $\xi = 0.1$ , and two choices of the parameter four-plet  $C_\alpha, C_{\phi\alpha}, C_\beta, C_{\phi\beta}$ . This value for  $\xi$  is consistent with  $\mathcal{O}(\xi^2)$ -calculations, while the  $C_i$ -parameter choices correspond to keeping only one term in  $\phi(t)$ , thus its derivative preserves the same sign and therefore  $w_{eff}$  behaves like in 4D quintessence (upper curve) or in 4D phantom (lower curve) paradigms.

In fig. 2 we depict  $w_{eff}$  versus  $z$  for four choices of the parameter-group  $\xi, C_\alpha, C_{\phi\alpha}, C_\beta, C_{\phi\beta}$ . In this case we are interested in acquiring a sign-flip of  $\dot{\phi}$ , and thus without loss of generality we keep the plus sign in  $\alpha$ -solution in (53) and the minus sign in  $\beta$ -solution in (54). As we observe,  $w_{eff}$  crosses the phantom divide from above to below in the

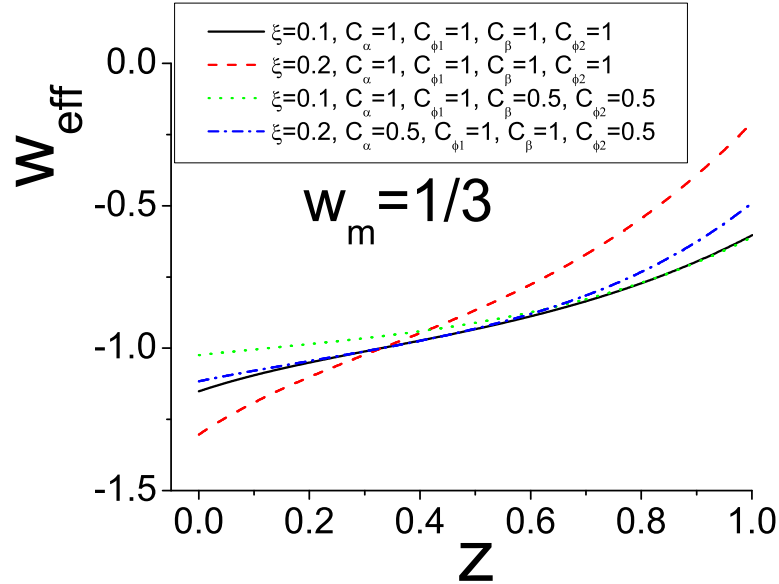


FIG. 2: (Color online)  $w_{eff}$  versus  $z$ , for  $w_m = 1/3$ . The values of the parameters  $\xi, C_\alpha, C_{\phi\alpha}, C_\beta, C_{\phi\beta}$  are shown in the inset.

recent cosmological past, as required by observations. This behavior is qualitatively independent of the values of the parameters  $\xi, C_\alpha, C_{\phi\alpha}, C_\beta, C_{\phi\beta}$ , however the precise values of  $z_c$  (redshift at the crossing) and of  $w_{eff0}$  (present value of  $w_{eff}$ ) do depend on them. Thus, we see that in the simple, but quite general, example of the present section, the

$-1$ -crossing can be achieved relatively easily, without a restriction to a small area of the parameter space. This feature makes braneworld models with a non-minimally coupled phantom bulk field a good candidate for the description of the current universe acceleration.

## V. CONCLUSIONS

In this work we examine general braneworld models, with a non-minimally coupled phantom bulk field and arbitrary brane and bulk matter contents. Imposing the low-energy, that is late-time assumptions, and performing the calculations up to first order in the non-minimal coupling  $\xi$ , we provide a general relation that connects the equation-of-state parameter  $w_{eff}$  of the  $4D$  effective dark energy, with the values of the phantom field  $\phi$  and its derivative at the location of the brane-universe. For  $\xi = 0$ ,  $w_{eff}$  is always  $-1$  and the dark energy of the brane-universe behaves like a cosmological constant, as expected. However, when the non-minimal coupling is switched on, the brane-universe's dark energy acquires a dynamical nature.

In particular  $w_{eff}$  is related to  $\xi$ , to the scale factor and its derivative, and to  $\phi$  and its derivative on the brane (relation (37)). Thus, if  $\phi\dot{\phi}$  preserves the same sign during the cosmological evolution, then  $w_{eff}$  remains always on the same side of the phantom divide and the universe's dark energy behaves like a quintessence or conventional phantom. On the other hand, if  $\phi\dot{\phi}$  experiences a sign-flip at some particular time, then  $w_{eff}$  crosses  $-1$ , in agreement with observations. This behavior is general and, up to first order in  $\xi$ , it is independent of the bulk and brane matter contents. That is, the non-minimal coupling provides a mechanism for an indirect “bulk-brane interaction”, through gravity. Furthermore, the crossing behavior appears without the need of special ansatzes for the various quantities, or of the restriction to specific, small areas of the parameter space.

In the present model, the  $-1$ -crossing arises from a single, non-minimally coupled phantom bulk field, and thus it is more economical than the multi-field models of conventional cosmology. This behavior was already known to hold in  $4D$  single-field, non-minimally coupled cosmology, but only for specific cases [20]. The fact that it is not only maintained in the higher-dimensional (and thus closer to a fundamental theory of nature) brane cosmology, but it appears for a much larger solution sub-class and parameter sub-space, is definitely a novel feature and an advantage of the model. This result provides an additional argument for the significance of the non-minimal coupling between gravity and scalar fields, for the description of nature. However, the possible quantization difficulties of such phantom scenarios is an open problem and needs further investigation.

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